Dog Vaccination and Quarantine, A Mathematical Approach on Rabies

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Background

- Rabies is a viral infection that is transmitted by animals.
- Rabies is prominent in the Asian and African Region.
- World Health Organization states that 99% of Rabies is spread through domestic dogs.
- The vaccine is administered for animals and humans before and after exposure.
- The Health and Human Society recommends getting the shot if you are at high risk for exposure which lasts for 2 years.
- Typical Veterinarians administer a vaccine for animals to last for 3 years.

Background

Source: Hiddenchina.net

- Our research looks at south eastern China.
- This area has reported almost 20,000 cases of Rabies in between 2005 and 2011 [8].
- The climate in this region is tropical which can be compared to Guam's climate.

Motivation

- Center for Disease Control states that Rabies is very deadly if left untreated.
- If the disease is allowed to incubate in the system the virus affects the brain which is almost 100% lethal.
- Rabies is a rare disease to find in the United States but is still present.
- Guam has a large population of both stray and domesticated dogs.
- Information on Rabies is not well known throughout the Marianas and is important since many residents are pet owners.

Goal

- Use mathematical methods to model the behavior of Rabies and utilize Game Theory to provide individuals a strategy to combat the disease.

Parameters for the model

Para.	Interpretation	Value(per month)	Source
B	Dog birth population	2.34×10^5	Estimation[6]
λ	Dog loss rate of immunity	$\frac{1}{6}$	Assumption[6]
m	Dog natural mortality rate	0.0064	Assumption[6]
a	The baseline contact rate	9.9×10^{-8}	Estimation[6]
b	The magnitude of forcing	0.41	Estimation [6]
μ	Dog disease-related death	1	MOHC [9]
β	Transmission rate between Dogs	1.4×10^{-7}	Estimation[6]
k	Dog vaccination rate	varies	variable
q	Dog quarantine rate	varies	variable

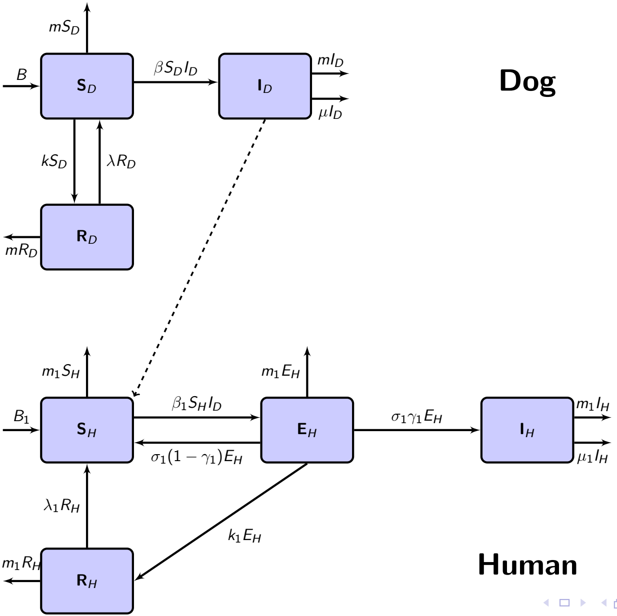
Parameters for the model (cont.)

Para.	Interpretation	Value (per month)	Source
B_1	Human birth population	1.34×10^6	NBSC [10]
β_1	Transmission rate between dogs and humans	2.96×10^{-11}	Estimation[6]
λ_1	Human loss rate of immunity	$\frac{1}{6}$	ChinaCDC [1]
i_1	Human incubation period	2	ChinaCDC [1]
σ_1	Human incubation rate $\frac{1}{\mathbf{i}_1}$	$\frac{1}{2}$	ChinaCDC [1]
γ_1	Clinical outcome rate of exposed humans	0.5	AnshanCDC [4]
m_1	Human natural mortality rate	0.00057	NBSC [10]
a_1	The baseline contact rate	2.41×10^{-11}	Estimation [6]
b_1	The magnitude of forcing	0.23	Estimation [6]
k_1	Human vaccination rate	0.54	MOHC [9]
μ_1	Human disease-related death rate	1	MOHC [9]

Model Compartments

- $\boxed{S_D}$ - Compartment of Susceptible dogs
- $\boxed{I_D}$ - Compartment of Infected dogs
- $\boxed{R_D}$ - Compartment of Recovered dogs
- $\boxed{S_H}$ - Compartment of Susceptible humans
- $\boxed{E_H}$ - Compartment of Exposed humans
- $\boxed{I_H}$ - Compartment of Infected humans
- $\boxed{R_H}$ - Compartment of Recovered humans

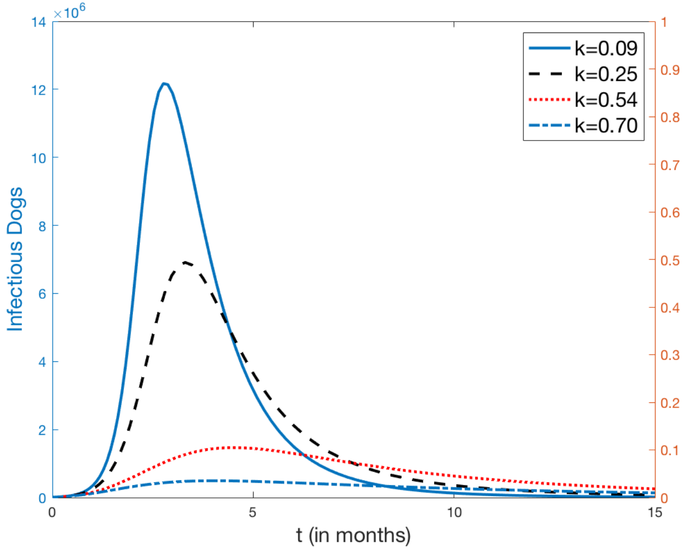
Rabies Model



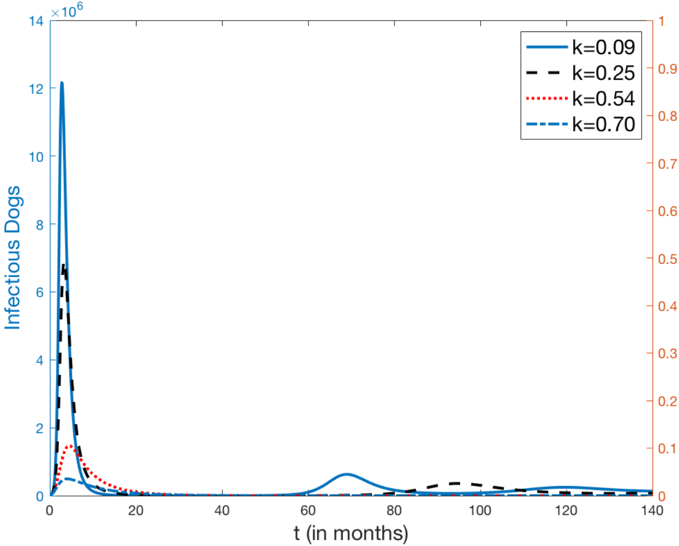
Differential Equations

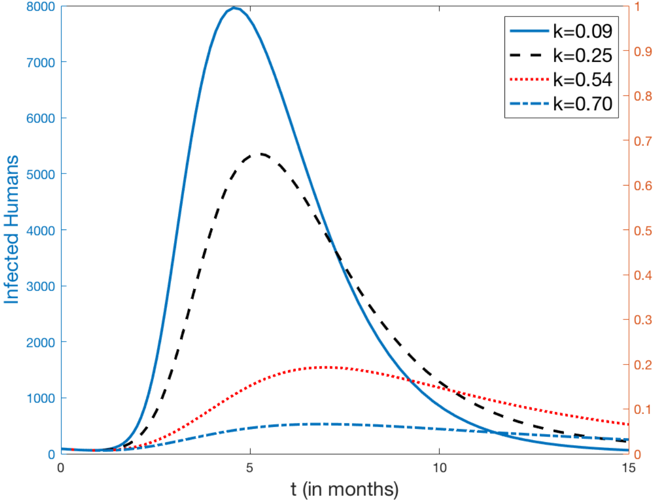
$$\frac{dS_D}{dt} = B + \lambda R_D - \beta S_D I_D - (m + k)S_D$$
$$\frac{dI_D}{dt} = \beta S_D I_D - (m + \mu)I_D$$
$$\frac{dR_D}{dt} = kS_D - (m + \lambda)R_D$$
$$\frac{dS_H}{dt} = B_1 + \lambda_1 R_H + \sigma_1(1 - \gamma_1)E_H - m_1 S_H - \beta_1 S_H I_D$$
$$\frac{dE_H}{dt} = \beta_1 S_H I_D - (m_1 + \sigma_1 + k_1)E_H$$
$$\frac{dI_H}{dt} = \sigma_1 \gamma_1 E_H - (m_1 + \mu_1)I_H$$
$$\frac{dR_H}{dt} = k_1 E_H - (m_1 + \lambda_1)R_H$$
$$N_D(t) = S_D(t) + I_D(t) + R_D(t)$$
$$N_H(t) = S_H(t) + E_H(t) + I_H(t) + R_H(t)$$

Vaccination Effects in the Model

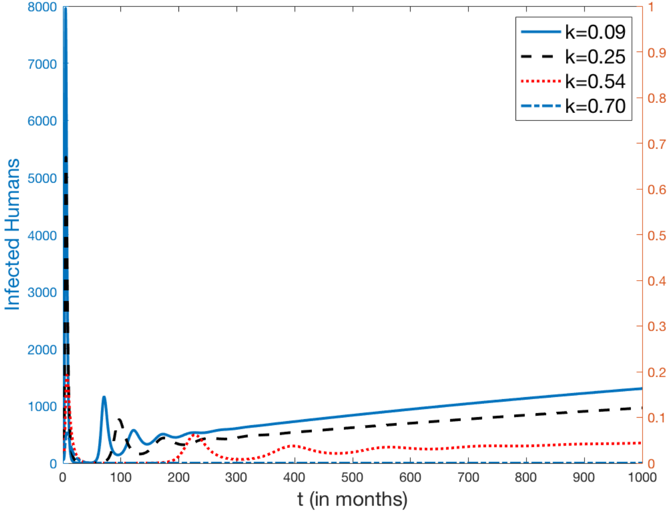


Vaccination Effects in the Model (cont.)

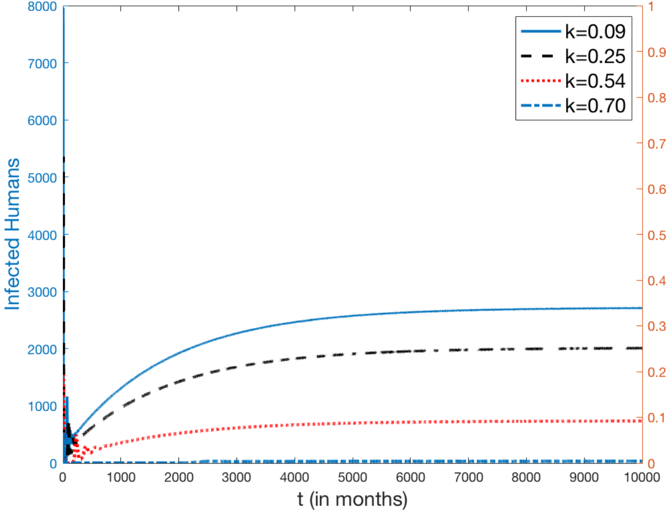



k=0.09
k=0.25
k=0.54
k=0.70
Infected Humans
t (in months)
0
1000
2000
3000
4000
5000
6000
7000
8000
5
10
15
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Vaccination Effects in the Model (cont.)



Vaccination Effects in the Model (cont.)



Basic Reproduction Number R_0

Definition (Basic Reproduction Number R_0)

The Basic Reproduction Number, R_0, is the number of secondary infections produced by a typical case of an infection in a population that is totally susceptible. (Health Knowledge 2009 [5])

Theorem (Driessche , Watmough 2001 [3])

1. *If $R_0 < 1$ then the disease is locally asymptotically stable.*
2. *If $R_0 > 1$ then the disease is unstable.*

We find our Basic Reproduction Number by using the Next Generation Matrix Method.

Basic Reproduction Number

- Our basic reproduction number is the following, where V represents vaccination.

$$R_0^v = \frac{S_D^0 \beta}{(m+\mu)} \qquad where \qquad S_D^0 = \frac{B(m+\lambda)}{m(m+\lambda+k)}$$

- By setting the vaccination rate to zero (k=0), we induce a R_0^{NV} with no vaccine

$$R_0^{NV} = \frac{B\beta}{m(m+\mu)}$$

Herd Immunity k_{HI}

Definition (Herd Immunity k_{HI})

Herd Immunity threshold, k_{HI}, is the proportion of the population that needs to be immune in order for an infectious disease to become stable (Health Knowledge 2009 [5])

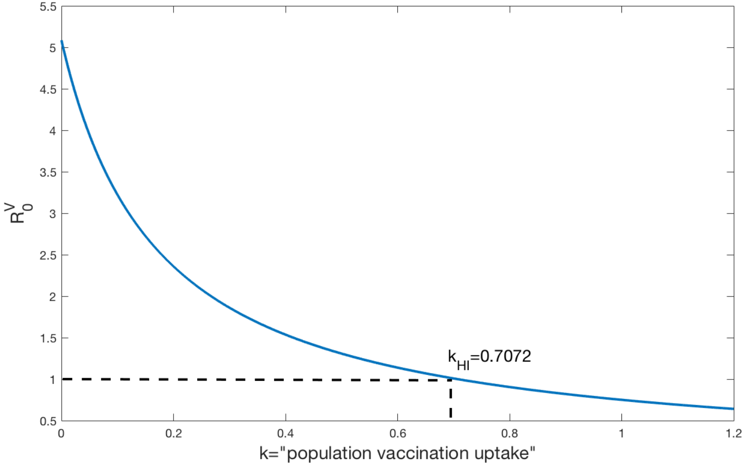
The k_{HI} is found by setting $R_0^V = 1$ and solving for k.

$$k_{HI} = (R_0^{NV} - 1)(m + \lambda)$$

$$\text{where} \quad R_0^{NV} = \frac{B\beta}{m(m + \mu)}$$

Running the data through the equation gives us a Herd Immunity of 70.72%.

Basic Reproduction Number Graph



Basic Reproduction Number Sensitivity

We obtained the Basic Reproduction Number:

$$R_0^V = \frac{B\beta(m+\lambda)}{m(m+\mu)(m+\lambda+ⓚ)}$$

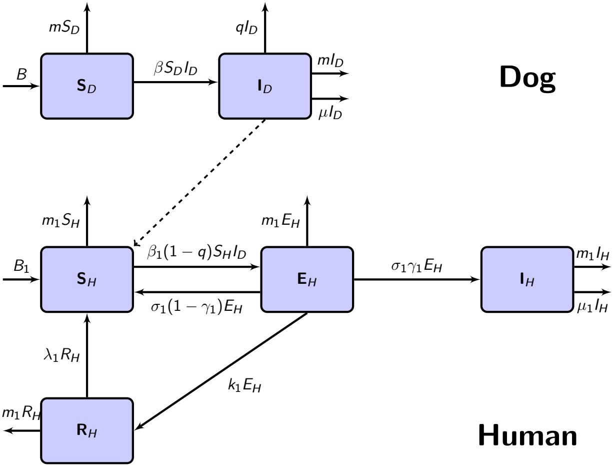
From here we can see that R_0^V is sensitive to the variable "k" (Vaccination rate) which is the only parameter that can be controlled thus we can say:

- **If $k < k_{HI}$ then $R_0^V > 1$ and $I_D = I_D^*$**
  i.e. the disease will continue to spread out.
- **If $k > k_{HI}$ then $R_0^V < 1$ and $I_D = I_D^0$**
  i.e. the disease will eventually die out.

Quarantine

- Another method of battling Rabies is to quarantine.
- By quarantining infectious dogs they are taken out of the model in which they will eventually die off since Rabies is a quite lethal disease.
- For this method we set k=0.
- Add q as a rate coming out of I_D.
- Let β vary.
- Replace β_1 with $\beta_1(1-q)$.
- "q" will be the quarantine rate where it is between [0,1].

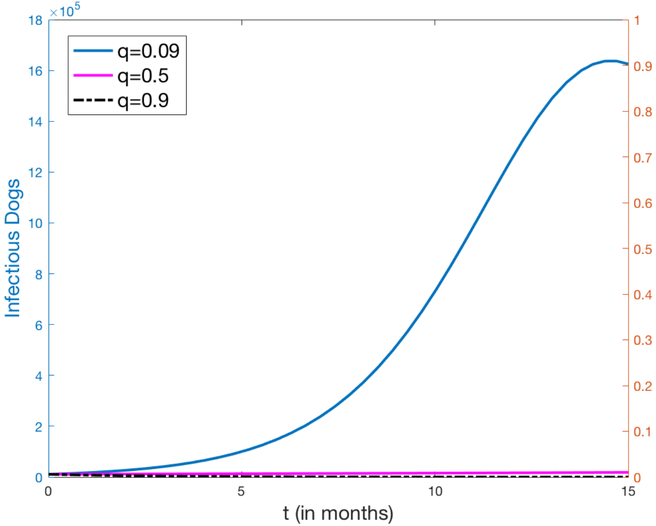
Rabies model - Quarantine



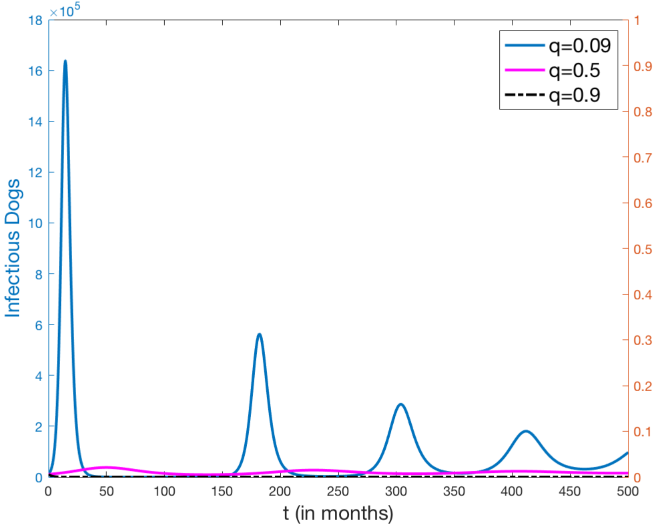
Differential Equations for Quarantine

$$
\begin{aligned}
\frac{dS_D}{dt} &= B - \beta S_D I_D - mS_D \\
\frac{dI_D}{dt} &= \beta S_D I_D - (m + \mu + q) I_D \\
\frac{dS_H}{dt} &= B_1 + \lambda_1 R_H + \sigma_1 (1 - \gamma_1) E_H - m_1 S_H - \beta_1 (1 - q) S_H I_D \\
\frac{dE_H}{dt} &= \beta_1 (1 - q) S_H I_D - (m_1 + \sigma_1 + k_1) E_H \\
\frac{dI_H}{dt} &= \sigma_1 \gamma_1 E_H - (m_1 + \mu_1) I_H \\
\frac{dR_H}{dt} &= k_1 E_H - (m_1 + \lambda_1) R_H \\
N_D(t) &= S_D(t) + I_D(t) + R_D(t) \\
N_H(t) &= S_H(t) + E_H(t) + I_H(t) + R_H(t)
\end{aligned}
$$

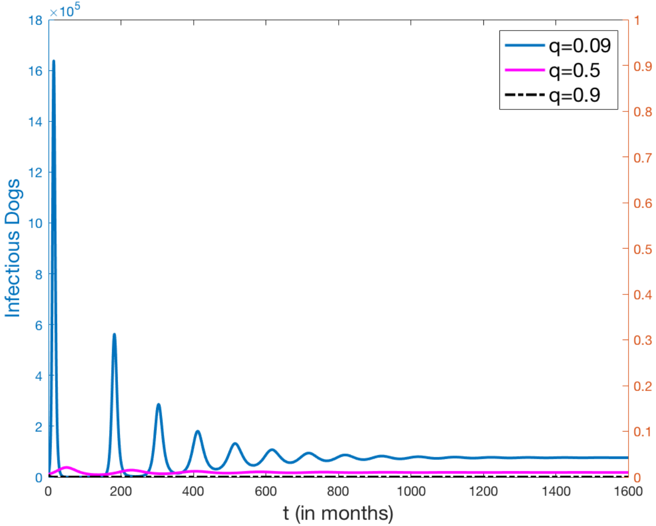
Quarantine Effects in the Model



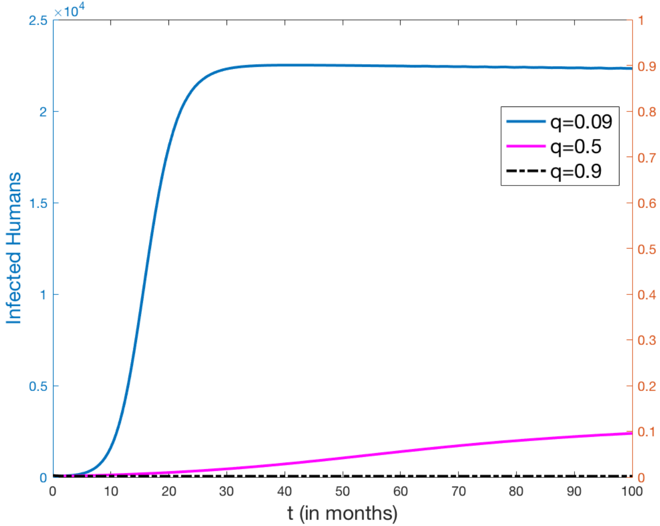
Quarantine Effects in the Model cont.



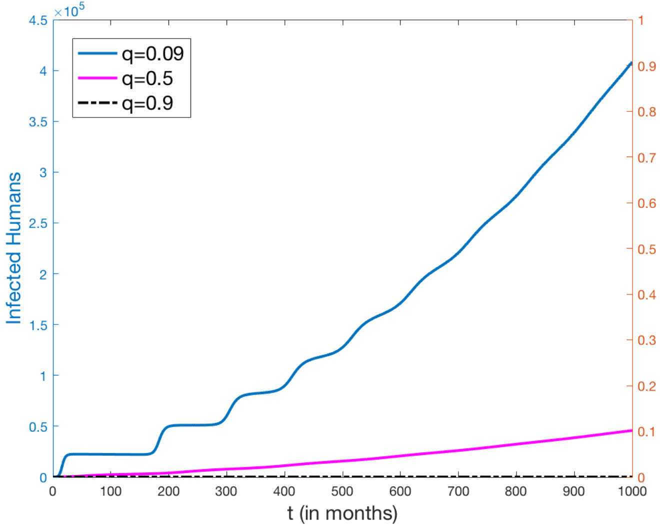
Quarantine Effects in the Model cont.



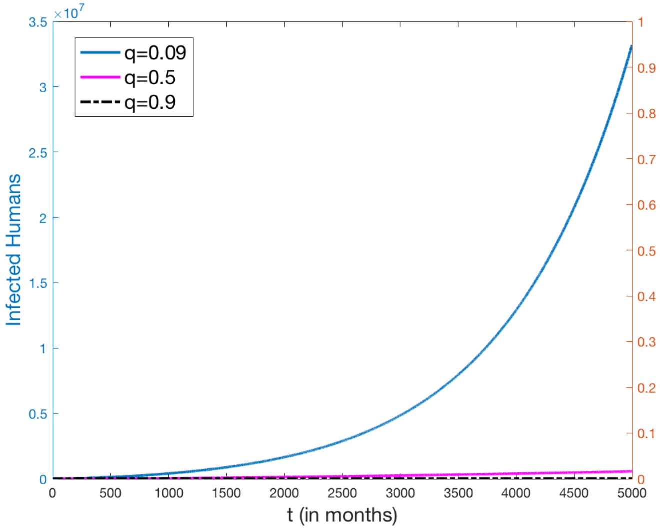
Quarantine Effects in the Model cont.



Quarantine Effects in the Model cont.



Quarantine Effects in the Model cont.



Endemic Equations - Quarantine

By setting the differential equations to zero, we assume that the infectious compartments are not zero since the disease is endemic. Where by solving for each compartment results in the following:

$$
\begin{aligned}
S_D^* &= \frac{(m + \mu + q)}{\beta} \\
I_D^* &= \frac{B\beta - m(m + \mu + q)}{\beta(m + \mu + q)} \\
S_H^* &= \frac{(m_1 + \sigma_1 + k_1}{\beta_1(1 - q)I_D^*} \\
E_H^* &= \frac{B_1\beta_1(1 - q)(m_1 + \lambda_1)I_D^*}{(m_1 + \lambda_1)[(\beta_1(1 - q)I_D^* + m_1)(m_1 + \sigma_1 + k_1) - \sigma_1(1 - \gamma_1)] - \beta_1(1 - q)\lambda_1 k_1 I_D^*} \\
I_H^* &= \frac{\sigma_1\gamma_1 E_H^*}{m_1 + \mu_1} \\
R_H^* &= \frac{k_1 E_H^*}{m_1 + \lambda_1}
\end{aligned}
$$

Basic Reproduction Number - Quarantine

Once again, we utilize the next generation matrix method to induce R_0^Q.

$$\mathscr{F} = \begin{bmatrix} \beta S_D I_D \\ \beta_1(1-q)S_H I_D \\ 0 \end{bmatrix}, \quad \mathscr{V} = \begin{bmatrix} (m+\mu+q)I_D \\ (m_1+\sigma_1+k_1)E_H \\ (m_1+\mu)I_H - \sigma_1\gamma_1 E_H \end{bmatrix}$$

$$F = \begin{bmatrix} \beta S_D^0 & 0 & 0 \\ \beta_1(1-q)S_H^0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} m+\mu+q & 0 & 0 \\ 0 & m_1+\sigma_1+k_1 & 0 \\ 0 & -\sigma_1 k_1 & m_1+\mu_1 \end{bmatrix}$$

$$FV^{-1} = \begin{bmatrix} \frac{\beta S_D^0}{m+\mu+q} & 0 & 0 \\ \frac{\beta_1 S_H^0}{m_1+\sigma_1+k_1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_0^Q = \rho\left(FV^{-1}\right)$$

$$R_0^Q = \frac{\beta S_D^0}{m+\mu+q} \quad \text{where} \quad S_D^0 = \frac{B}{m}$$

Herd Immunity - Quarantine

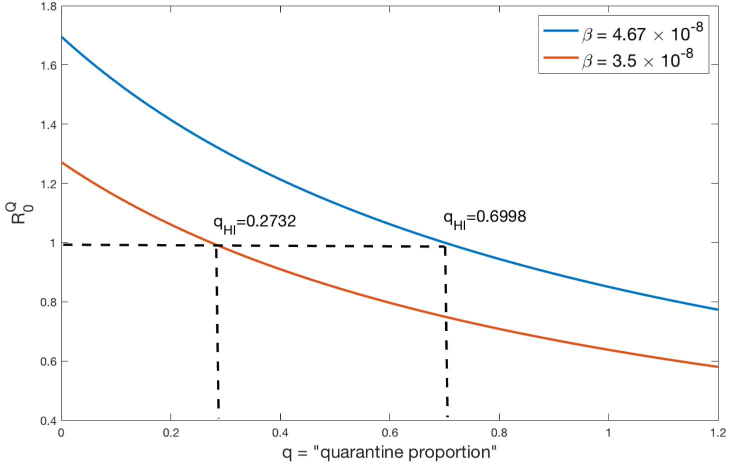
The q_{HI} is found by setting $R_0^Q = 1$ and solving for q.

$$1 = \frac{B\beta}{m(m + \mu + q)}$$

$$q_{HI} = \frac{B\beta - m(m + \mu)}{m}$$

- Running the data through the equation gives us a Herd Immunity of 69.98% where β is 4.67×10^{-8}, a third of our original β.
- Similarly, by setting β to 3.5×10^{-8}, a fourth of our original β, we receive a Herd Immunity of 27.32%.

Basic Reproduction Number

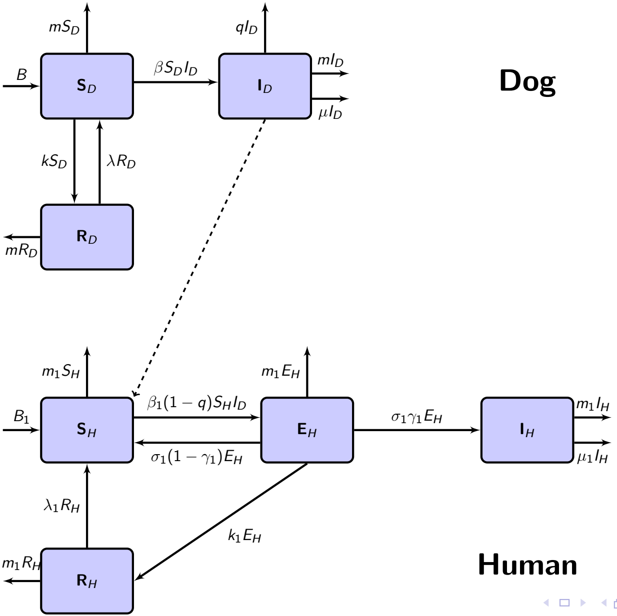


Basic Reproduction Number R_0^Q

Based on the graph we interpret the following behavior to occur.

1. **If** $q < q_{HI}$ **then that** $R_0^Q > 1$ **and** $I_D = I_D^*$
   i.e. the disease will continue to spread out.
2. **If** $q > q_{HI}$ **then that** $R_0^Q < 1$ **and** $I_D = I_D^0$
   i.e. the disease will eventually die out.

Rabies Model - Vaccination and Quarantine



Differential Equations

$$\frac{dS_D}{dt} = B + \lambda R_D - \beta S_D I_D - (m+k)S_D$$
$$\frac{dI_D}{dt} = \beta S_D I_D - (m+\mu)I_D$$
$$\frac{dR_D}{dt} = kS_D - (m+\lambda)R_D$$
$$\frac{dS_H}{dt} = B_1 + \lambda_1 R_H + \sigma_1(1-\gamma_1)E_H - m_1 S_H - \beta_1(1-q)S_H I_D$$
$$\frac{dE_H}{dt} = \beta_1(1-q)S_H I_D - (m_1+\sigma_1+k_1)E_H$$
$$\frac{dI_H}{dt} = \sigma_1\gamma_1 E_H - (m_1+\mu_1)I_H$$
$$\frac{dR_H}{dt} = k_1 E_H - (m_1+\lambda_1)R_H$$
$$N_D(t) = S_D(t) + I_D(t) + R_D(t)$$
$$N_H(t) = S_H(t) + E_H(t) + I_H(t) + R_H(t)$$

Combination Equations

By solving the equations for the combination of both methods we receive:

Disease Free:

$$E_0 = (S_D^0, I_D^0, R_D^0, S_H^0, E_H^0, I_H^0, R_H^0)$$
$$= (\frac{B(m+\lambda)}{m(m+\lambda+k)}, 0, \frac{kB}{m(m+\lambda+k)}, \frac{B_1}{m_1}, 0, 0, 0)$$

Endemic:

$$I_D^* = \frac{B\beta(m+\lambda) - m(m+\lambda+k)(m+\mu+q)}{\beta(m+\lambda)(m+\mu+q)}$$

Basic Reproduction Number:

$$R_0 = \frac{B\beta(m+\lambda)}{m(m+\mu+\textcircled{q})(m+\lambda+\textcircled{k})}$$

Game Theory - Vaccination

Definition (Game Theory)

Game Theory is the analysis of a situation involving conflicting interests in terms of gains and losses among opposing players (Merriam-Webster 2018 [7]).

Let us set up the game:

- **Goal**: Propose the best strategy for an individual dog owner to take.
- **Player** - Individual Dog Owner (P).
- **Strategies**
  - Vaccinate (V) - Vaccine the dog.
  - Non-Vaccinate(NV) - Do not vaccine the dog.

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Game Situations

We have two situations at hand when playing the game.

Situation 1: Disease Free

Condition:	$k > k_{HI}$
Action:	The dog owner will not vaccinate their dog, P(NV).

Situation 2: Endemic

Condition:	$k < k_{HI}$
Action:	Naturally a person would expect to vaccinate but we must consider the cost.

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Game Theory - Vaccination

When the disease is in the endemic situation, using Game Theory, we look at the expected payoffs for the decision (Bauch [2]).

$$E(V,k) = -C_{VD} - \pi_V C_{ID} C_{IH} - C_{VH}$$
$$E(NV,k) = -\pi_{NV} C_{ID} C_{IH} - C_{VH}$$

Symbol	Notation
E(V,k)	Expected Payoff for Vaccinating a Dog
E(NV,k)	Expected Payoff for Not Vaccinating a Dog
C_{VD}	Cost of Vaccinating a Dog
C_{VH}	Cost of Vaccinating a Human
C_{ID}	Cost of an Infectious Dog
C_{IH}	Cost of an Infected Human
π_{NV}	Probability of a Susceptible Dog becoming infectious and the probability of a susceptible human becoming infected
π_V	Probability of a Vaccinated Dog becoming infectious and the probability of a susceptible human becoming infected

From these expected payoffs we receive the following:

- If $E(V,k) < E(NV,k)$,
  then the decision would be to not take the vaccine for the cost of the vaccine outweighs the cost of not vaccinating.
- If $E(V,k) > E(NV,k)$,
  then the decision would be to take the vaccine for the cost of having an infectious dog outweighs the cost of the vaccine.

Now, we analyze the interesting case when $E(V,k) = E(NV,k)$.

Nash Equilibrium

Definition (Nash Equilibrium)

The Nash Equilibrium is the optimal strategy in a game where a player will benefit the most regardless of the opponents strategy (Investopedia 2018).

Our Nash Equilibrium is defined as k_{NE} where we set

$$E(V,k) = E(NV,k)$$
$$-C - \pi_V - C_{VH} = -\pi_{NV} - C_{VH}$$
$$\text{where} \quad C = \frac{C_{VD}}{C_{IH}C_{ID}}$$
$$-C - (\pi_{RS})(\pi_{NV}) = -\pi_{NV}$$
$$(1 - \pi_{RS})(\pi_{NV}) = C$$

π_{RS} is defined as the probability of a dog going from Recovered (comparment R) to Susceptible (compartment S) which can be computed,

$$\frac{\text{The transfer from R into S}}{\text{The total transfers out of R}}.$$

Nash Equilibrium - Vaccination

$$C = \left(\frac{m}{m+\lambda}\right)\left(\frac{\beta I_D^*}{\beta I_D^* + m + k}\right)\left(\frac{\beta_1 I_D^*}{\beta_1 I_D^* + m_1}\right)\left(\frac{\sigma_1\gamma_1}{\sigma_1 + m_1 + k_1}\right)$$

where $I_D^* = \frac{B\beta(m+\lambda) - m(m+\lambda+k)(m+\mu)}{\beta(m+\lambda)(m+\mu)}$

Thus we solve for k_{NE} as a function of C.

$$\begin{aligned}&[\beta_1 mxC\lambda - m^2]k_{NE}^2 \\ &\quad -[xC(\beta_1 my - \beta_1 y\lambda + \beta_1 z\lambda) + 2mz - 2\beta_1 ym]k_{NE} \\ &\qquad + [xyC(\beta_1 y - \beta_1 z + w) - (\beta_1 y^2 - 2\beta_1 yz + z^2)] = 0\end{aligned}$$

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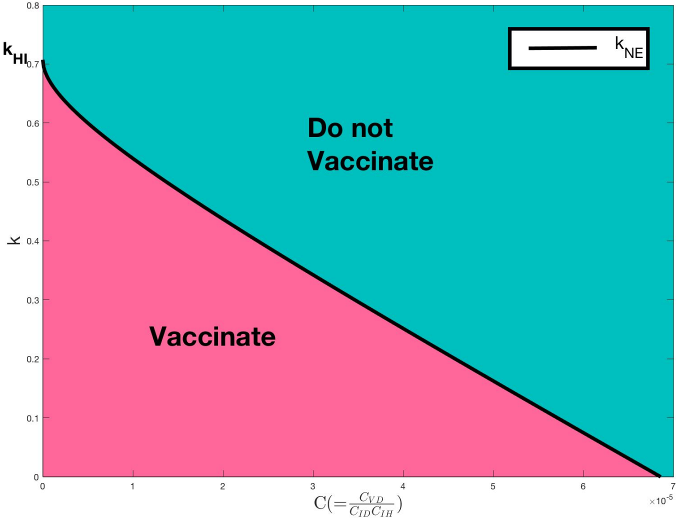
Nash Equilibrium - Vaccination

$$k_{NE} = \frac{vyzC + \beta_1 vxyC - 2\beta_1 xy}{2vy^2C - 2\beta_1 y^2} + \frac{\sqrt{(vyzC + \beta_1 vxyC - 2\beta_1 xy)^2 + 4(vy^2C - \beta_1 y^2)(\beta_1 vxzC + vwzC - \beta_1 x^2)}}{2vy^2C - 2\beta_1 y^2}$$

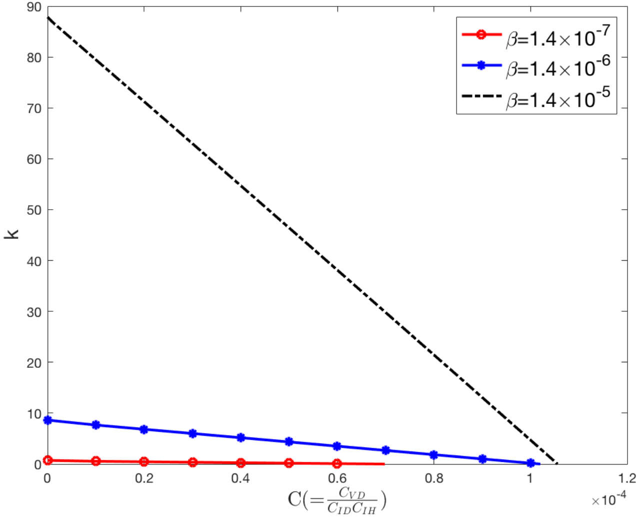
where

$$x = B\beta(m+\lambda) - m(m+\mu)(m+\lambda)$$
$$y = m(m+\mu)$$
$$z = B\beta(m+\lambda)$$
$$w = \beta m_1(m+\lambda)(m+\mu)$$
$$v = \frac{(m+\lambda)(m_1+\sigma_1+k_1)}{m\sigma_1\gamma_1}$$

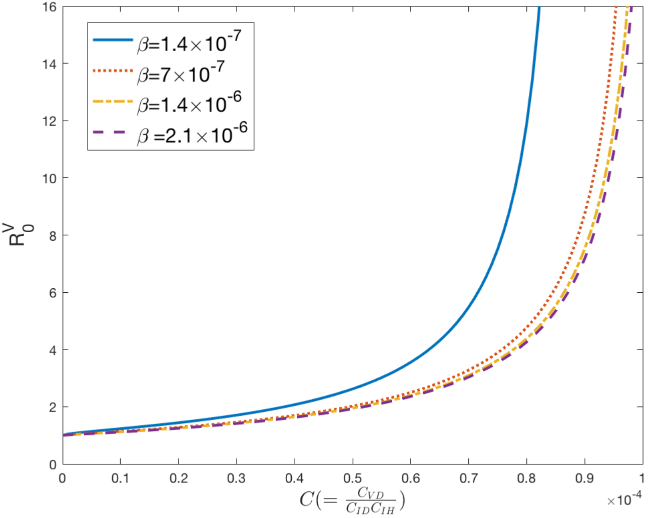
Nash Equilibrium - Vaccine



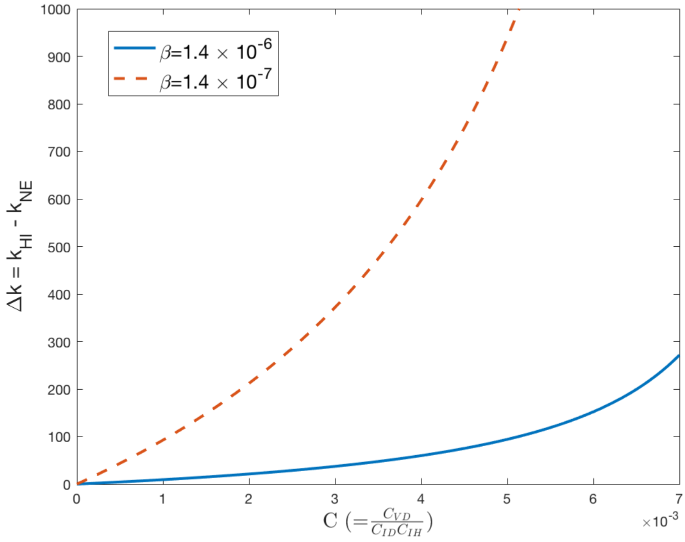
Nash Equilibrium - Vaccine



Basic Reproduction Number with Cost



Difference between K_{HI} and K_{NE}



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Game Theory - Quarantine

Similarly to vaccinating we set up the game for quarantine.

- **Goal**: Propose the best strategy for an individual dog owner to take.
- **Player** - Individual Dog Owner (P)
- **Strategies**
  - Quarantine (Q) - Quarantine the dog.
  - Non-Quarantine(NQ)- Do not quarantine the dog.

Again we are left with two situations that will occur when playing the game.

Situation 1: Disease Free

Condition:	$q > q_{HI}$
Action:	The dog owner will not quarantine their dog, P(NQ).

Situation 2: Endemic

Condition:	$q < q_{HI}$
Action:	Naturally a person would expect to quarantine but we must consider the cost.

Game Theory - Quarantine

When looking at the Endemic situation we look at the following payoffs:

$$E(Q,q) = -C_Q - \pi_Q C_{ID} C_{IH} - C_{VH}$$
$$E(NQ,q) = -\pi_{NQ} C_{ID} C_{IH} - C_{VH}$$

Symbol	Notation
E(Q,q)	Expected Payoff for quarantining of an infectious dog
E(NQ,q)	Expected Payoff for not quarantining of an infectious dog
C_Q	Cost of quarantine
π_{NQ}	Probability of a Susceptible Human becoming infected by a not quarantined infectious dog
π_Q	Probability of a Susceptible Human becoming infected by a quarantined infectious dog

Expected Payoff

From these expected payoffs we receive the following:

- If $E(Q, q) < E(NQ, q)$ then the decision would be to not quarantine for the cost of the quarantine outweighs the cost of not vaccinating.
- If $E(Q, q) > E(NQ, q)$ then the decision would be to quarantine for the cost of having an infectious dog outweighs the cost of the quarantine.

Now we analyze the interesting case when $E(Q, q) = E(NQ, q)$.

Quarantine Nash Equilibrium

When we set the expected payoffs equal we solve for q:

$$-C_Q - \pi_Q C_{ID} C_{IH} = -\pi_{NQ} C_{ID} C_{IH}$$

$$\overline{C} = \pi_{NQ} \quad \text{where} \quad \overline{C} = \frac{C_Q}{C_{IH} C_{ID}}$$

$$\overline{C} = (\pi_{S_H E_H})(\pi_{E_H I_H})$$

$$\overline{C} = \left(\frac{\beta_1 (1-q) I_D^*}{\beta_1 (1-q) I_D^* + m_1} \right) \left(\frac{\sigma_1 \gamma_1}{m_1 + \sigma_1 + k_1} \right)$$

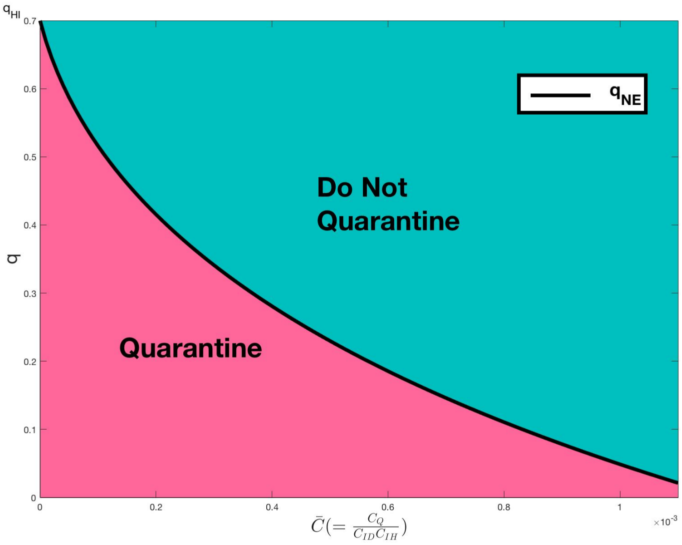
Quarantine Nash Equilibrium

Thus by solving the equation for q_{NE} in terms of $\overline{C}$

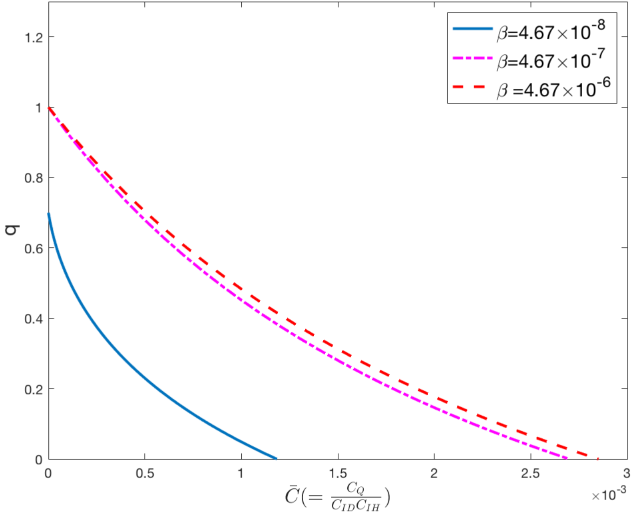
$$q_{NE} = \frac{\sqrt{[\beta_1(m+y)(1-x\overline{C})]^2 - 4\beta_1 m(x\overline{C}-1)(\beta_1 vxz\overline{C} + vwz\overline{C} - \beta_1 x^2)}}{2\beta_1 m(x\overline{C}-1)} - \frac{m+y}{2}$$

where
$$x = \frac{m_1 + \sigma_1 + k_1}{\sigma_1 \gamma_1}$$
$$y = B\beta - m(m+\mu)$$
$$z = \beta m_1(m+\mu)$$

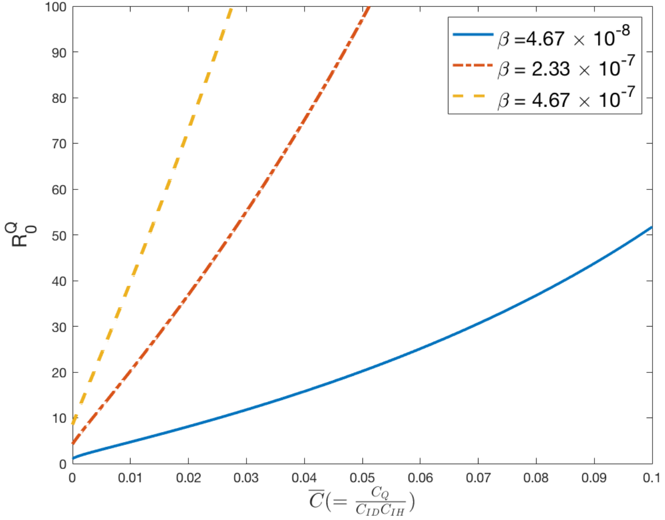
Nash Equilibrium - Quarantine



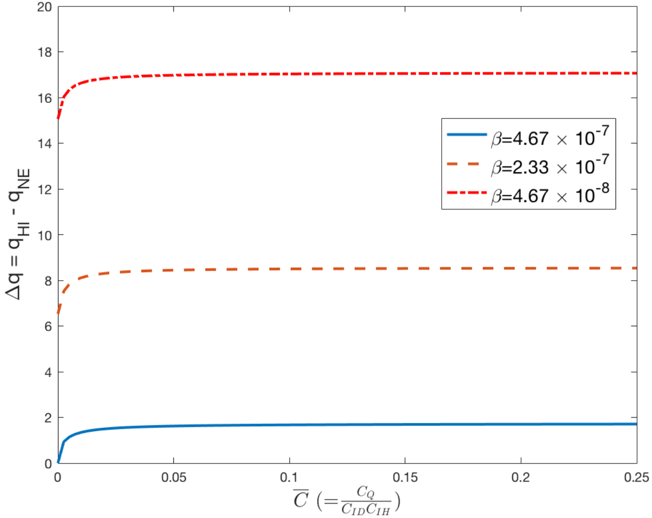
Nash Equilibrium - Quarantine



Basic Reproduction Number - Quarantine



Difference between q_{HI} and q_{NE}



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Combination of Quarantine and Vaccination

From the methods of combating Rabies mentioned earlier, we assume that one method was not existent. Here we analyze the efforts of both methods to eliminate Rabies.

- **Goal**: Decide which strategy is best when the cost of both strategies are available.
- **Player**: Individual Dog Owner
- **Strategy**:
  - Quarantine (Q)
  - Vaccinate (V)

We find q_{NE} and k_{NE} as functions of the other strategy.

Re-evaluating the Strategies

$$C = \left(\frac{m}{m+\lambda}\right)\left(\frac{\beta I_D^*}{\beta I_D^* + m + k}\right)\left(\frac{\beta_1(1-q)I_D^*}{\beta_1(1-q)I_D^* + m_1}\right)\left(\frac{\sigma_1\gamma_1}{\sigma_1 + m_1 + k_1}\right)$$

$$\overline{C} = \left(\frac{\beta_1(1-q)I_D^*}{\beta_1(1-q)I_D^* + m_1}\right)\left(\frac{\sigma_1\gamma_1}{m_1 + \sigma_1 + k_1}\right)$$

where $I_D^* = \frac{B\beta(m+\lambda) - m(m+\lambda+k)(m+\mu+q)}{\beta(m+\lambda)(m+\mu+q)}$

Where we solve for k_{NE} and q_{NE} for C and $\overline{C}$ respectively.

Nash Equilibrium

$$k_{NE}(q) = \frac{vyzC + \beta_1(1-q)vxyC - 2\beta_1(1-q)xy}{2vy^2C - 2\beta_1(1-q)y^2} + \frac{\sqrt{(vyzC + \beta_1(1-q)vxyC - 2\beta_1(1-q)xy)^2 + 4(vy^2C - \beta_1(1-q)y^2)(\beta_1(1-q)vxzC + vwzC - \beta_1(1-q)x^2)}}{2vy^2C - 2\beta_1(1-q)y^2}$$

where

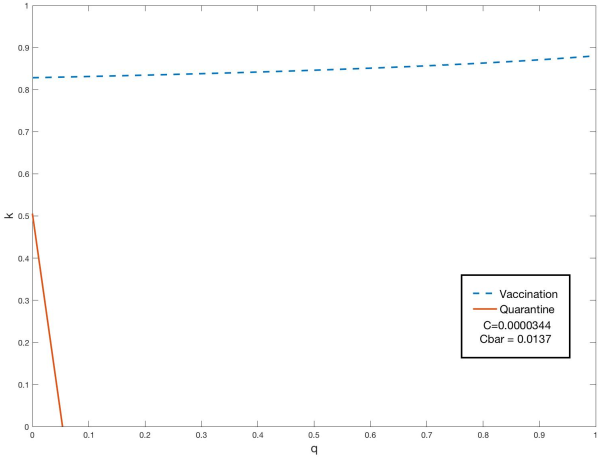
$$\begin{aligned} x &= B\beta(m+\lambda) - m(m+\mu)(m+\lambda) \\ y &= m(m+\mu) \\ z &= B\beta(m+\lambda) \\ w &= \beta m_1(m+\lambda)(m+\mu) \\ v &= \frac{(m+\lambda)(m_1+\sigma_1+k_1)}{m\sigma_1\gamma_1} \end{aligned}$$

$$q_{NE}(k) = \frac{(\beta_1 y + \beta_1 xz\overline{C} - \beta_1 x - \beta_1 yz\overline{C} - vz\overline{C})}{2\beta_1 y(1 - z\overline{C})} + \frac{\sqrt{(\beta_1 y + \beta_1 xz\overline{C} - \beta_1 x - \beta_1 yz\overline{C} - vz\overline{C})^2 - 4(\beta_1 y(1 - z\overline{C}))(\beta_1 x - \beta_1 xz\overline{C} + wz\overline{C})}}{2\beta_1 y(1 - z\overline{C})}$$

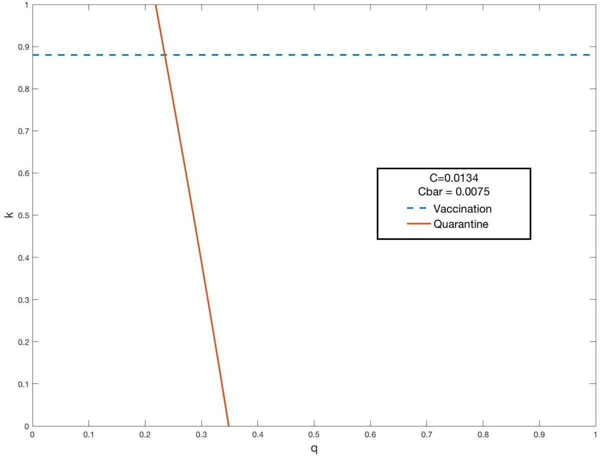
where

$$\begin{aligned} x &= B\beta(m+\lambda) + m(m+\lambda+k)(m+\mu) \\ y &= m(m+\lambda+k) \\ z &= \frac{m_1+\sigma_1+k_1}{\sigma_1\gamma_1} \\ w &= \beta m_1(m+\lambda)(m+\mu) \\ v &= \beta m_1(m+\lambda) \end{aligned}$$

Vaccination Dominant



Mixed Strategy



Conclusion

- We can see that by comparing the two strategies when in terms of relative cost, Vaccination is more sensitive. When comparing the cost of vaccine and quarantine, we can recommend vaccination.
- We also see that when the disease is more deadly people are willing to pay for the vaccine. If the disease is more deadly and the only option is to quarantine, people would not quarantine their dog as it is costly.

Future Research

- Further study into the relation between C and $\overline{C}$, to better predict the dominant and mixed strategies of vaccination and quarantine
- Algebraic analysis to look at the conditions between the strategy for Quarantine and Vaccination.

Q & A

Questions?

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